

Appendix from T. R. Haaland et al., “Differential Allocation Revisited: When Should Mate Quality Affect Parental Investment?” (Am. Nat., vol. 190, no. 4, p. 000)

Part A. Analytical Model

As described in the main text, the optimal level of female investment u_f^* can be found by solving the equation

$$\frac{\partial W}{\partial u_f} = \frac{\partial b_t(u_f, q_m)}{\partial u_f} + \alpha \frac{\partial c(u_f, q_m)}{\partial u_f} \frac{\partial \bar{b}_{t+1}(\tilde{u}_f, c(u_f, q_m))}{\partial c} = 0, \quad (\text{A1})$$

where the bar in $\partial \bar{b}_{t+1} / \partial c$ denotes the average over future male qualities $\tilde{q}_{m,i}$, weighted by the probability that the female pairs with each quality male, p_i . Qualities of future males do not depend on female investment in our model and thus can be treated as constants when calculating a female’s best response to current male quality q_m .

A1. Absence or Presence of Differential Allocation

We see that differential allocation (DA) is expected when equation (A1) holds and $\partial^2 W / \partial q_m \partial u_f \neq 0$. In the benefit scenarios, the current male’s quality does not affect the female’s future fecundity, that is, $c(u_f, q_m) = c(u_f)$, nor does the female’s investment in the current time step affect her future mating opportunities. Therefore, we are interested only in whether the first term in the optimization equation (A1), $\partial b_t(u_f, q_m) / \partial u_f$, depends on q_m .

In the benefit-elevation scenario (fig. 1A), male quality only contributes additively to female current fecundity; we can write $b_t(u_f, q_m) = b_t(u_f) + g(q_m)$, where g is a positive function that reflects the strength with which the current male contributes to the female’s offspring. Then, $\partial b_t(u_f, q_m) / \partial u_f = b'_t(u_f)$, which does not depend on q_m , and we do not expect DA to arise in a benefit-elevation scenario.

In the benefit-slope scenario (fig. 1B), female investment and male quality determine offspring fitness in a multiplicative fashion, so we write $b_t(u_f, q_m) = b_t(u_f) \times g(q_m)$ so that increasing male quality effectively accelerates the returns on female investment. Consequently, the $\partial b_t(u_f, q_m) / \partial q_m$ remains a function of both u_f and q_m , and DA should therefore occur.

The benefit-position scenario (fig. 1C) has the same expression for finding u_f^* as benefit-slope, the only mathematical difference between the scenarios being that benefit-position assumes decelerating returns as male quality increases, $\partial b'_t(u_f, q_m) / \partial q_m < 0$. Therefore, we expect DA here as well (but see sec. A2).

In the cost scenarios, the current male’s quality does not affect the female’s current offspring fitness benefits, $b_t(u_f, q_m) = b_t(u_f)$, so only the dependence of the second term in equation (A1) on q_m is relevant.

For the cost-elevation scenario, male quality affects $c(u_f, q_m) = c(u_f) - k(q_m)$, where the positive function $k(q_m)$ reflects how strongly male quality reduces the female cost function, as a current male of high quality is assumed to reduce future costs to females. Whether such a scenario produces DA depends on the shape of the future benefit function, since future fecundity b_{t+1} is affected by q_m through $k(q_m)$, and its derivative may also do so. If it depends linearly on $k(q_m)$, for example,

$$b_{t+1}(\tilde{u}_f, \tilde{q}_{m,i}, c(u_f) - k(q_m)) = \tilde{u}_f - c(u_f) + k(q_m) + \tilde{q}_{m,i},$$

then the derivative with respect to u_f becomes

$$b'_{t+1}(\tilde{u}_f, \tilde{q}_{m,i}, c(u_f) - k(q_m)) = -c'(u_f)$$

and DA does not occur. However, for a nonlinear benefit function, such as the four-parameter logistic function we use below, the derivative b'_{t+1} is a function of both u_f and q_m , so cost-elevation results in DA, despite the fact that the marginal costs of current reproduction $c'(u_f, q_m) = c'(u_f)$ are independent of the quality q_m of the current male. Rather, the nonlinearity in the benefit function will allow any absolute change in costs to affect the marginal benefits of future reproduction b'_{t+1} , and DA is thus favored.

In the cost-slope and cost-position scenarios, female investment and male quality affect the female’s cost function multiplicatively, so we write $c(u_f, q_m) = c(u_f) \times k(q_m)$. The marginal cost c' then depends on both u_f and q_m , so DA will always be expected, whether the benefit function is linear or nonlinear. The only mathematical difference between the

two scenarios is the extra assumption in cost-position that better male quality pushes the female down on her cost function and costs increase less steeply further down the cost function, so we have $\partial c'(u_f, q_m)/\partial q_m < 0$.

A2. Direction of DA

We use second-order conditions to investigate direction of DA (positive or negative), calculating female investment u_f as the female's best response $r(q_m)$ to current male quality q_m , after which we substitute for $u_f \equiv r(q_m)$ in equation (A1) and take the derivative of $(\partial W/\partial u_f)|_{u_f=r(q_m)}$ with respect to q_m , which will allow us to solve for $r'(q_m)$. We then obtain

$$\begin{aligned} \frac{\partial}{\partial q_m} \left[\frac{\partial W}{\partial u_f} \Big|_{u_f=r(q_m)} \right] &= \frac{\partial}{\partial q_m} \left[\frac{\partial b_t(r(q_m), q_m)}{\partial u_f} + \alpha \frac{\partial c(r(q_m), q_m)}{\partial u_f} \frac{\partial \bar{b}_{t+1}(\tilde{u}_f, c(r(q_m), q_m))}{\partial c} \right] \\ &= r'(q_m) \frac{\partial^2 b_t(r(q_m), q_m)}{\partial r \partial u_f} + \frac{\partial^2 b_t(r(q_m), q_m)}{\partial q_m \partial u_f} \\ &\quad + \alpha \left[r'(q_m) \frac{\partial^2 c(r(q_m), q_m)}{\partial r \partial u_f} + \frac{\partial^2 c(r(q_m), q_m)}{\partial q_m \partial u_f} \right] \frac{\partial \bar{b}_{t+1}(\tilde{u}_f, c(r(q_m), q_m))}{\partial c} \\ &\quad + \alpha \frac{\partial c(r(q_m), q_m)}{\partial u_f} \left[r'(q_m) \frac{\partial c(r(q_m), q_m)}{\partial r} + \frac{\partial c(r(q_m), q_m)}{\partial q_m} \right] \frac{\partial^2 \bar{b}_{t+1}(\tilde{u}_f, c(r(q_m), q_m))}{\partial c^2} = 0. \end{aligned} \tag{A2}$$

Solving for $r'(q_m)$ and dropping other obvious notation, we obtain

$$\begin{aligned} r'(q_m) &= \left(\frac{\partial^2 b_t}{\partial q_m \partial u_f} + \alpha \frac{\partial^2 c}{\partial q_m \partial u_f} \frac{\partial \bar{b}_{t+1}}{\partial c} + \alpha \frac{\partial c}{\partial u_f} \frac{\partial c}{\partial q_m} \frac{\partial^2 \bar{b}_{t+1}}{\partial c^2} \right) \\ &\quad / \left(- \frac{\partial^2 b_t}{\partial r \partial u_f} - \alpha \frac{\partial^2 c}{\partial r \partial u_f} \frac{\partial \bar{b}_{t+1}}{\partial c} - \alpha \frac{\partial c}{\partial u_f} \frac{\partial c}{\partial r} \frac{\partial^2 \bar{b}_{t+1}}{\partial c^2} \right), \end{aligned} \tag{A3}$$

where the survival probability α is a positive constant and hence does not affect the sign of the expression above. We show below that the denominator is always positive (see sec. A2b). The sign of the numerator (and hence the direction of DA) depends, however, on the specific scenario.

A2A. THE SIGN OF THE NUMERATOR OF EQUATION (A3)

Benefit-Slope: Positive DA. Because costs c depend only on u_f and not on q_m , all terms in the numerator are 0, except the first. We have $\partial^2 b_t/\partial q_m \partial u_f > 0$, as increasing male quality increases the slope of b_t with respect to female investment (see fig. 1B). Consequently, we have $\partial r(q_m)/\partial q_m > 0$ (positive DA).

Benefit-Position: Negative DA. Again, because costs c depend only on u_f and not on q_m , all terms in the numerator are 0, except the first. We have $\partial^2 b_t/\partial q_m \partial u_f < 0$, as increasing male quality now decreases the slope of b_t with respect to female investment (see fig. 1C). Consequently, we have $\partial r(q_m)/\partial q_m < 0$ (negative DA).

All Cost Scenarios: Positive DA. Regarding the first term in the numerator, we have $\partial^2 b_t/\partial q_m \partial u_f = 0$, because male quality has no effect on b_t for all cost scenarios (see fig. 1D–1F). Regarding the second term, we have $\partial \bar{b}_{t+1}/\partial c < 0$, as increasing costs reduce future fecundity. Next, we have $\partial^2 c/\partial q_m \partial u_f \leq 0$, as costs are linear in u_f for the cost elevation and cost slope scenario, in which case $\partial^2 c/\partial q_m \partial u_f = 0$, while $\partial^2 c/\partial q_m \partial u_f < 0$ holds for the cost position scenario, as increasing male quality reduces the marginal increase of female costs with increased female investment. Hence, the second term is positive. Regarding the third term, we have $\partial^2 \bar{b}_{t+1}/\partial c^2 \leq 0$, as future female benefits decrease either linearly or in an accelerating fashion with increasing costs. In addition, $\partial c/\partial u_f > 0$, as costs increase with increasing female investment, and $\partial c/\partial q_m < 0$, as costs decrease with increasing male quality. Hence, also the third term is positive. Consequently, as we have already shown in section A1 that the numerator is nonzero for the cost scenarios, the numerator is thus strictly positive. Hence, all cost scenarios result in $\partial r(q_m)/\partial q_m > 0$ (positive DA).

A2B. THE DENOMINATOR OF EQUATION (A3) IS POSITIVE

We have $\partial^2 b_t / \partial r \partial u_f \leq 0$, because current fecundity is either linear in u_f or a decelerating function of u_f (and hence r). Regarding the second term in the numerator, we have $\partial^2 c / \partial r \partial u_f \geq 0$ because costs increase either linearly or acceleratingly with female investment. In addition, we have $\partial \bar{b}_{t+1} / \partial c < 0$ because increasing costs reduce future fecundity. With regards to the third term, we have $\partial c / \partial u_f > 0$ and $\partial c / \partial r > 0$ because costs increase with increasing levels of female investment, while we have $\partial^2 \bar{b}_{t+1} / \partial c^2 \leq 0$, because future female benefits decrease either linearly or in an accelerating fashion with increasing costs. Ignoring the degenerate case where the denominator is 0 (in which case $\partial r(q_m) / \partial q_m$ has no solution), the denominator is thus positive.

A3. Cost of Current Reproduction Affects Female Survival Instead of Future Fecundity

So far, we have assumed that costs of current reproduction $c(u_f, q_m)$ reduce a female's amount of energy that can be allocated to future reproduction. Here we consider an alternative scenario in which costs of current reproduction affect a female's survival to the next time step $\alpha \equiv \alpha[c(u_f, q_m)]$ instead and where future fecundity $b_{t+1} \equiv b_{t+1}(\tilde{u}_f, \tilde{q}_{m,j})$ is now affected only by a female's future investment \tilde{u}_f and future male quality $\tilde{q}_{m,j}$. We then have

$$\frac{\partial W}{\partial u_f} = \frac{\partial b_t(u_f, q_m)}{\partial u_f} + \frac{\partial c(u_f, q_m)}{\partial u_f} \frac{\partial \alpha[c(u_f, q_m)]}{\partial c} \bar{b}_{t+1}(\tilde{u}_f) = 0, \quad (\text{A4})$$

and again we assess when $\partial^2 W / \partial q_m \partial u_f \neq 0$. In the benefit scenarios, we have $c(u_f, q_m) = c(u_f)$, so that again only $\partial b_t(u_f, q_m) / \partial u_f$ needs to be evaluated (which has already been done in sec. A1). Hence, regardless of whether costs affect future fecundity or survival, DA occurs in the benefit-slope and benefit-position scenarios.

In the cost scenarios, we have $b_t(u_f, q_m) = b_t(u_f)$, so that only the second part in equation (A4) affects DA, and dropping obvious notation,

$$\frac{\partial^2 W}{\partial q_m \partial u_f} = \left(\frac{\partial^2 c}{\partial q_m \partial u_f} \frac{\partial \alpha}{\partial c} + \frac{\partial c}{\partial u_f} \frac{\partial^2 \alpha}{\partial c^2} \frac{\partial c}{\partial q_m} \right) \bar{b}_{t+1}.$$

For the cost-elevation scenario, we have $c(u_f, q_m) = c(u_f) - k(q_m)$, so that $\partial^2 c / \partial q_m \partial u_f = 0$. Hence, DA occurs only when costs affect survival in a nonlinear fashion (i.e., $\partial^2 \alpha / \partial c^2 \neq 0$), which is analogous to the finding in section A1 that the future fecundity function should be nonlinear. For the cost-slope and cost-position scenarios, this restriction does not apply and DA occurs also when costs affect survival in a linear fashion. Hence, we conclude that there are no differences between the survival cost and the future fecundity cost scenarios in terms of DA.

Regarding the direction of DA, we find

$$r'(q_m) = \left(\frac{\partial^2 b_t}{\partial q_m \partial u_f} + \frac{\partial^2 \alpha}{\partial c^2} \frac{\partial c}{\partial q_m} \frac{\partial c}{\partial u_f} \bar{b}_{t+1} + \frac{\partial \alpha}{\partial c} \frac{\partial^2 c}{\partial q_m \partial u_f} \bar{b}_{t+1} \right) / \left(-\frac{\partial^2 b_t}{\partial r \partial u_f} - \frac{\partial^2 \alpha}{\partial c^2} \frac{\partial c}{\partial r} \frac{\partial c}{\partial u_f} \bar{b}_{t+1} - \frac{\partial \alpha}{\partial c} \frac{\partial^2 c}{\partial r \partial u_f} \bar{b}_{t+1} \right), \quad (\text{A5})$$

where again the denominator is positive (see below). For the benefit scenarios, costs c depend only on u_f and not on q_m , so all terms in the numerator are 0, except the first. Hence, results collapse to those in section A2a, with positive DA in a benefit-slope scenario and negative DA in a benefit-position scenario. For the cost scenarios we again always find positive DA: as in section A2a, we have $\partial^2 b_t / \partial q_m \partial u_f = 0$. Regarding the second term, we have $\partial^2 \alpha / \partial c^2 \leq 0$ (costs either linearly or acceleratingly decrease survival), while $\partial c / \partial q_m < 0$ and $\partial c / \partial u_f > 0$ (see sec. A2a). Hence, the second term is always positive. Regarding the third term, we have $\partial \alpha / \partial c < 0$ (costs decrease survival) and $\partial^2 c / \partial q_m \partial u_f \leq 0$ (see sec. A2a), so that also the third term is positive. Note that $\bar{b}_{t+1} \geq 0$ since fecundity is a nonnegative number.

The denominator of equation (A5) is positive. As set out in section A2b, we have $\partial^2 b_t / \partial r \partial u_f \leq 0$, $\partial c / \partial r > 0$, $\partial c / \partial u_f > 0$, and $\partial^2 c / \partial r \partial u_f \geq 0$. In addition, $\partial^2 \alpha / \partial c^2 \leq 0$ and $\partial \alpha / \partial c < 0$ (costs either linearly or acceleratingly decrease survival) and $\bar{b}_{t+1} \geq 0$, which together result in a positive denominator.

Part B. Details of the Stochastic Dynamic Model

Each time step in the model represents one breeding season or opportunity in which the female may choose how large a reproductive investment u_f she makes in one offspring. Offspring fitness is represented by $b(u_f)$, and the female's costs are $c(u_f)$. Costs reduce female condition or energetic state X , but reproductive investment cannot lower female state to below the limit x_{\min} .

There are $N_m = 3$ discrete classes of male qualities, q_m , in the population, and at each time step a female pairs with a male of random quality, following a specified probability distribution, P . We consider male quality as if each class of male provides the female with a specific cost function $c(u_f, q_m)$ or a specific offspring fitness function $b(u_f, q_m)$, depending on the scenario. Coding of the program was inspired by Roff (2010, chap. 6) and entails the biological assumption that females are evolutionarily aware (but possibly not conscious) of the distribution of qualities of males they may meet in the future and thus their probability of meeting them. The entire model was created in R, version 2.15.1 (R Core Team 2012), and the coding uses the additional package "abind" (Plate and Heiberger 2011). The code is freely available on Dryad (Haaland et al. 2017, <http://dx.doi.org/10.5061/dryad.qc561>).

The model is dimensioned so as to avoid female decisions early in life being affected by the possibility of terminal rewards at time T —that is, a mathematical solution that would mask the other realisms our model aims to reveal—and produces optimal investment decisions u for all combinations of time step, state, and male quality. In order to examine lifetime trajectories for individual females, we also ran forward simulations using the Markov chain Monte Carlo (MCMC) method (Clark and Mangel 2000), based on the optimal investments from the output of the stochastic dynamic model.

Optimization

The backward iteration produces three-dimensional matrices of optimal investments, $u_f^*(x, q_m, t)$, and fitness values given those optimal investments, $W^*(u_f^*, x, m, t)$. The fitness values W are made up of current and future fitness components. The current fitness component is the fitness of the offspring produced in that reproductive event with male type m , $b_i(u_f, q_m)$, and the future fitness component is the average expected fitness the female is able to gain in the rest of her lifetime if she acts optimally.

The backward iteration begins with calculating fitness values of the optimal choices for each state and mate type at end time step T , $W^*(u_f^*, x, m, t)$, and these are then used in the calculations at time step $T-1$. The future component of fitness here is then the probability of survival until the next time step, α , times the sum of fitness values $W^*(u_f^*, x, m, t)$ of each state that can be reached in the next time step, multiplied by the probabilities of getting to each of those states. This also depends on whether the female finds food, which increases her state by d , and this happens with a fixed probability λ . These values are also summed over each of the N_m classes of male that she can meet in the next time step, weighted over the probability P of meeting each class. Note that the summation variable for future males below is i , while the costs incurred (which affect state in the future fitness component) depend on the current male, m . As stated in the main text, at every time step back, fitness values W_u are calculated for each investment u :

$$W(u_f, x, q_m, t) = b_i(u_f, q_m) + \alpha \left[\sum_{i=1}^{N_m} p_i \{ \lambda W^*(x - c(u_f, q_m) + d, q_i, t + 1) + (1 - \lambda) W^*(x - c(u_f, q_m), q_i, t + 1) \} \right]$$

and the dynamic programming equation (Houston and McNamara 1999; Clark and Mangel 2000) determines W^* :

$$W^*(u^*, x, m, t) = \max_u \{ W(u, x, m, t) \}$$

so the u_f producing the highest W is saved as $u_f^*(x, q_m, t)$ in the matrices of optimal investments.

We ensured stabilization by making sure that fitness in the optimization is practically time independent for the early time steps (i.e., a difference between $W^*(u_f^*, x, q_m, t)$ and $W^*(u_f^*, x, q_m, t + 1)$ of below 10^{-9} for low t), as recommended by Houston and McNamara (1999). All scenarios were run with a range of different values of survival probabilities, α , as well as testing for effects arising with state-dependent mortality, that is, making survival probability a linearly increasing or curved decelerating function of state $\alpha(X)$.

Using different distributions of male qualities in the population, P , did not add any particular insights regarding DA to the results presented here, which are shown for $P = \{0.3, 0.4, 0.3\}$. Other distributions were examined, including one pessimistic scheme where most males are poor quality, $P = \{0.6, 0.2, 0.2\}$, and one optimistic one where there were very few poor-quality males, $P = \{0.1, 0.6, 0.3\}$. Beyond changing the absolute levels of investment, by shifting the

balance of current versus future expected fitness, the relative probabilities of meeting different-quality distributions of males did not greatly affect patterns of female reproductive investment.

Forward Simulations

The matrices produced by the backward iteration tell us what the optimal behaviors are once the female is in a given state with a given male quality at a given time but do not necessarily tell us which states or behaviors we might observe in a population of females following the optimal strategy. Therefore, we used this output of matrices to simulate lifetime trajectories of sequences of investment decisions for individual females, using the MCMC method (Clark and Mangel 2000). As stated above, the model is dimensioned so that the dynamic program finds no advantage for the female of saving reserves until the final time step, so fitnesses and optimal decisions are therefore time independent for low t . Under strong backward convergence (Houston and McNamara 1999) like this, forward simulations are less likely to reveal anything new, but we still ran them to check that the model was working sensibly, and we include the results in the supplementary material for completeness.

We simulated 1,000 female life spans in each scenario and tracked what state they were in, what quality of male they met, and how much they invested in reproduction at each time step. When they meet a male, they extract the correct element $u_f^*(x, q_m, t)$ from the best investment matrices from the dynamic model, depending on their state and current male. After investing this optimal investment, the costs of such investment are subtracted from their state (using probabilistic linear interpolation). We then use random number generation to determine whether the female survives until the next time step and whether she finds food (which adds to her energetic state) in the meantime. The initial female states were normally distributed around $x_{\max} * 2/3$ (x_{\max} is the maximum possible state) with a standard deviation of 1.

Model Scenarios

First we tested the series of basic male quality effects illustrated in figure 1, in order to clearly separate the independent effects of changing the elevation or the slope of the benefit ($b(u)$) or cost ($c(u)$) function. In these model runs, we used linear functions for the relationship that is affected by male quality (i.e., the focal function: linear $b(u)$ in benefit-elevation and benefit-slope; linear $c(u)$ in cost-elevation and cost-slope), changing either the elevation or the slope in turn, and curved functions for the relationship not affected by male quality (i.e., the background function: decelerating $b(u)$ [eq. (A6)] in cost-elevation and cost-slope; accelerating $c(u)$ [eq. (A7)] in benefit-elevation and benefit-slope; see ‘‘Classification of Male Quality Effects’’; fig. 1; table A2).

In addition, we demonstrated the effects of a positional shift on the $b(u)$ or $c(u)$ function, for benefit-position and cost-position scenarios, respectively. In these scenarios, we instead used the curved function as the focal one, because a positional shift on a linear function is indistinguishable from an elevation shift. For simplicity, these basic scenarios use linear functions as background functions, in order to avoid the unnecessary complexity of two curved functions at once (fig. 1C, 1F; table A2).

The curved functions used in these scenarios are

$$b(u) = a + \frac{k - a}{1 + e^{-g(u-v)}}, \quad (\text{A6})$$

$$c(u) = su + r^{su}, \quad (\text{A7})$$

where $a, k, g, v, r,$ and s are constants described in table A1. All of them may differ with mate quality in various scenarios, but the dependencies on m are omitted here for readability.

For the offspring fitness function (eq. [A6]), we use the upper part of a logistic function that is symmetrical around 0 (see table A1 for baseline parameter values), which is exclusively increasing and decelerating toward an asymptote. After completing the simple scenarios described above, we illustrate some biologically interesting issues using a more generally realistic sigmoid offspring fitness function. To do this, we reparameterized the logistic function in equation (A6) to include some or all of the accelerating part of the function as well, so the overall function becomes sigmoid over the range of possible investments. We first demonstrate the effect of a positional shift on the now sigmoid offspring fitness function (the sigmoid benefit-position scenario), and we then include model runs where male quality affects the parameters of the logistic function, providing sets of parameters that emulate different ecologically realistic scenarios (table A2; fig. 2).

Tables

Table A1: Description of variables and parameter values for the analytic and stochastic dynamic models

Variables	Parameter values
b	Benefits of reproductive investment for the female; the offspring fitness function
c	Costs of reproductive investment for the female; the female cost function
u_f	Female reproductive investment
q_m	Male quality
x	Female energetic state, with $x_{\max} = 30$ and $x_{\min} = 3$ and increments of 1; female dies if $x < x_{\min}$
α	Survival probability; set to .8 in baseline model runs
λ	Probability of finding food; set to .9 in baseline model runs
d	Energetic value of food; set to 2 in baseline model runs
T	Maximum life span; set to 100
m	Male qualities; $m = 1$ is a poor male, $m = 2$ is medium, $m = 3$ is good
N_m	No. discrete categories of male qualities; set to 3
P	Probability distribution of meeting the different male types; p_m is a component of vector P . We examined $P = \{.3, .4, .3\}$ (baseline), $P = \{.6, .2, .2\}$, and $P = \{.1, .6, .3\}$
a_m	Offspring fitness function lower asymptote; baseline -15
k_m	Offspring fitness function upper asymptote; baseline 15
g_m	Offspring fitness function growth rate; baseline $.2$
v_m	Offspring fitness function inflection point; baseline 0
r_m	Cost function shape parameter; baseline 1.7
s_m	Cost function slope parameter; baseline $.3$

Table A2: Description of model scenarios

Scenario	Model representation
Baseline scenarios with one linear and one curved function:	
Linear focal function and curved background function:	
Benefit-elevation	-1 or $+1$ added to $b(u)$ for poor and good males, respectively
Benefit-slope	Slope of $b(u)$ changed from 1 to .8 and 1.2 for poor and good males, respectively
Cost-elevation	$+1$ or -1 added to $c(u)$ for poor and good males, respectively
Cost-slope	Slope of $c(u)$ changed from .7 to .9 and .5 for poor and good males, respectively
Curved focal function and linear background function:	
Benefit-position	Investment u gives benefits $b(u - 1)$ and $b(u + 1)$ for poor and good males, respectively
Cost-position	Investment u gives costs $c(u + 1)$ and $c(u - 1)$ for poor and good males, respectively
Additional scenarios with sigmoid $b(u)$ and curved $c(u)$:	
Sigmoid benefit-position	$a = -1, g = .4, v = 6$
Juvenile survival	$a_m = \{-1, 3, 7\}, g = .4, k_m = \{14, 15, 16\}, v = 7$
High mating skew	$a = 0, g = .6, k_m = \{11, 15, 19\}, v = 8$

Note: Parameter values are in table A1 unless otherwise stated. Vectors of parameter values in braces represent values with male qualities $m = 1$ (poor), $m = 2$ (medium), and $m = 3$ (good).